Name:______________________________

Math 3113 Section 01 — *Practice Exam 2* — *November 19, 2019*

Follow the instructions for each question and show enough of your work so that I can follow your thought process. If I can't read your work or answer, you will receive little or no credit! On the actual exam the last page will have a list of matrices and their row reduced echelon or row echelon forms that you may or may not need.

1. Let

$$C^1(\mathbb{R}) = \{f(x) : f'(x) \text{ exists}\}$$

Show that $C^1(\mathbb{R})$ is a subspace of $C(\mathbb{R})$, the set of all continuous functions on the real line under the usual addition of functions and scalar multiplication.

2. Let

$$B(\mathbb{R}) = \{f(x) : f(a) = 0, \text{ for a fixed real number } a\}$$

Show that $B(\mathbb{R})$ is a subspace of $C(\mathbb{R})$, the set of all continuous functions on the real line under the usual addition of functions and scalar multiplication.

3. Let

$$S = \{t^2 + 1, t^2 + t, t + 1\}$$

Determine if the following set is a basis for P_2, the set of all polynomials of degree 2 or less.

4. Let

$$S = \{2t^2 + 1, 3t^2 - 4t, 4t + 9\}$$

Determine if the following set is a basis for P_2, the set of all polynomials of degree 2 or less.

5. Let $T : P_2 \to \mathbb{R}$ be a linear transformation defined by:

$$T(p(t)) = \int_0^1 (a_0 + a_1 t + a_2 t^2)\ dt$$

Compute $\mathrm{Ker}(T)$.

6. Let $T : P_2 \to P_1$ be a linear transformation defined by:

$$T(p(t)) = \frac{d}{dt}(a_0 + a_1 t + a_2 t^2)$$

Compute $\mathrm{Ker}(T)$.

7. Let A be an 10×10 matrix and let $d = \dim \mathrm{Ker}(A)$. Suppose d solves the following quadratic equation:

$$d^2 - 9d = 0$$

What are the possible dimensions of the columnspace to A? In either case is A invertible? (HINT: You will need the rank-nullity theorem)

8. Let A be an $m \times n$ matrix. Let $A^t A$ be nonsingular. Show that $\mathrm{rk}(A) = n$. (HINT: You will need the rank-nullity theorem)

9. Let A be an $n \times n$ matrix. Suppose there is no nonzero vector $\mathbf{x} \in \mathbb{R}^n$, such that $A\mathbf{x} = \mathbf{x}$. Show that $A - I_n$ is nonsingular. (HINT: You will need the rank-nullity theorem.)